ECE 344
Microwave Fundamentals

Lecture 08:
Power Dividers and Couplers
Part 1

Prepared By

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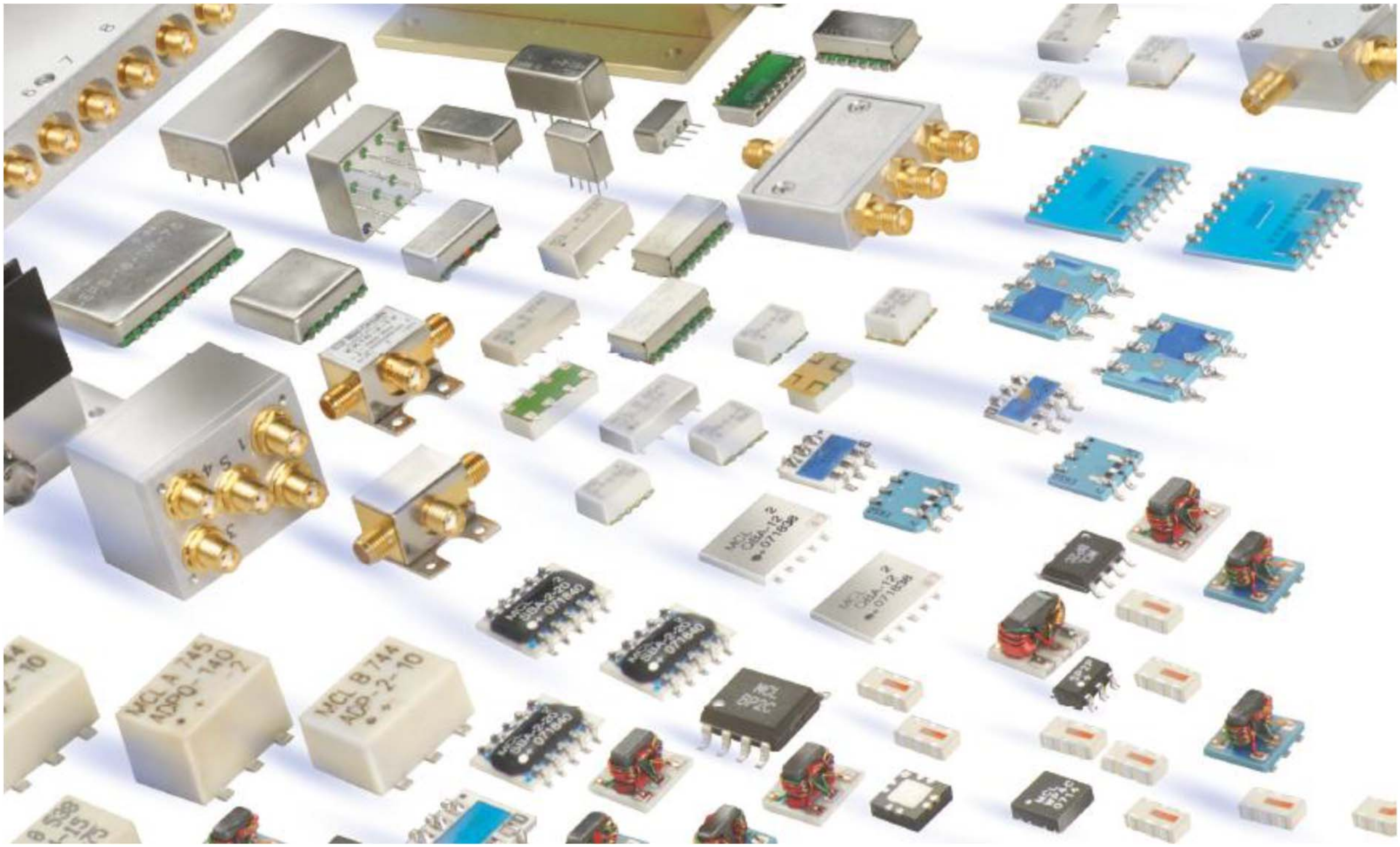
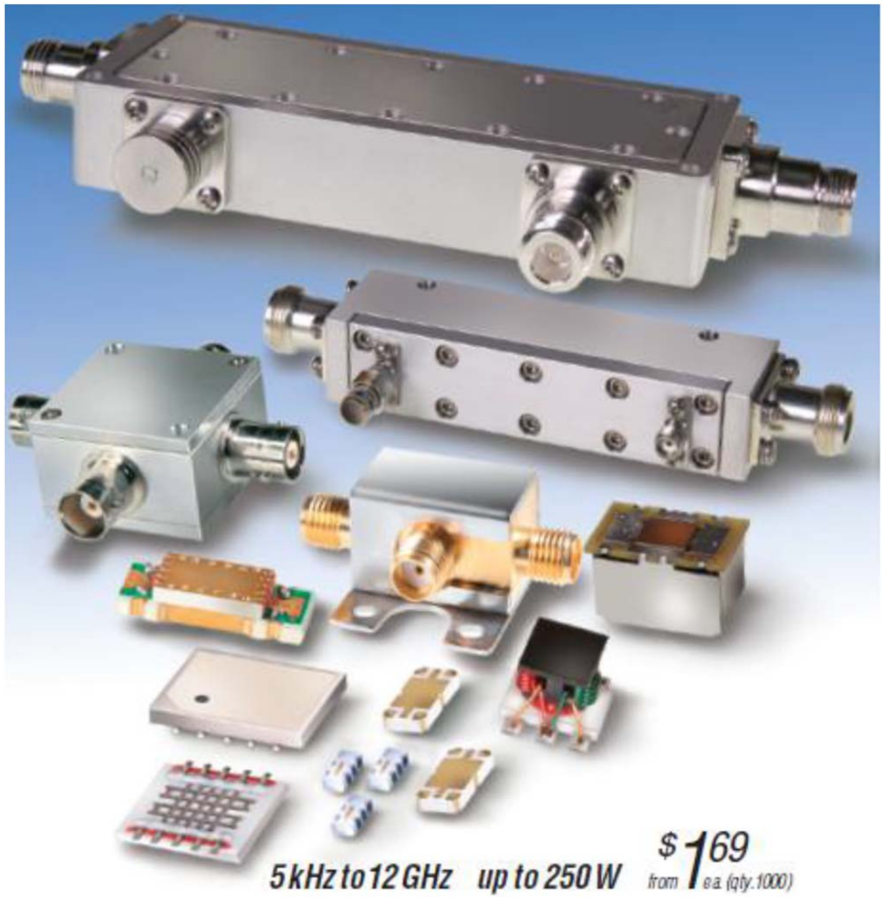
Microwave Devices



Microwave Devices



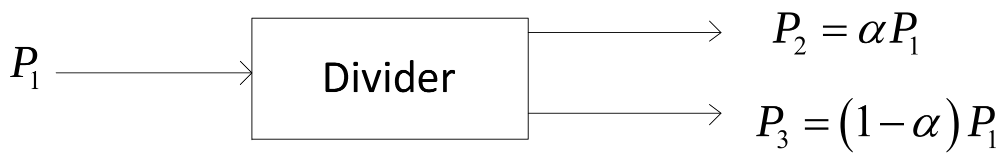
Power Dividers and Couplers

- Power dividers, combiners and directional couplers are passive structures that divide RF input power among several outputs or combine power from several inputs.
- Power Dividers and Combiners
  - ❑ Used to split input power into roughly equal outputs, or vice-versa.
- Directional Couplers
  - ❑ Used to sample a fraction of input power and/or to separate forward and reverse traveling waves.

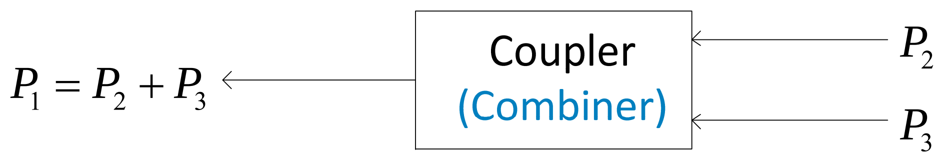
Power Dividers and Couplers

These are examples of a three-port network.

❖ A power divider is used to split a signal.



❖ A coupler/combiner is used to combine a signal.



- Goal: Distribute power from one input among several outputs, or combine power from several inputs to one output.
- Problems for RF and microwave designs
  - ❑ Impedance match
  - ❑ Isolation
  - ❑ Phase relationships among signals

Three Port Networks

General 3-port network:

$$[S] = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

Three Port Networks (cont.)

If all three ports are matched, and the device is reciprocal and lossless, we have:

$$[S]=\begin{pmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{pmatrix}$$

(The S matrix is also unitary.)

(There are three distinct values.)

This is not physically possible!

(see next slide)

Power Dividers and Couplers (cont.)

$$[S]=\begin{pmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{pmatrix}$$

← Unitary: not physically possible

Lossless $\Rightarrow$ $[S]$ is unitary

$$\Rightarrow \left|S_{12}\right|^2+\left|S_{13}\right|^2=1$$
$$\left|S_{12}\right|^2+\left|S_{23}\right|^2=1$$
$$\left|S_{13}\right|^2+\left|S_{23}\right|^2=1$$

← These cannot all be satisfied.

(If only one is nonzero, we cannot satisfy all three.)

$$S_{13}^*\,S_{23}=0$$
$$S_{12}^*\,S_{23}=0$$
$$S_{12}^*\,S_{13}=0$$

$\Rightarrow$ At least 2 of S_{13}, S_{12}, S_{23} must be zero.

(If only one is zero (or none is zero), we cannot satisfy all three.)

Unitary Matrix

- It can be done in an easy way:
  - The dot product of any column of [S] with the conjugate of that column gives unity.
  - The dot product of any column with the conjugate of a different column gives zero (orthogonal).

Circulators

Now consider a 3-port network that is non-reciprocal, with all ports matched, and is lossless:

$$\Rightarrow [S] = \begin{pmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{pmatrix}$$

(There are six distinct values.)

"Circulator"

⇧

These equations will be satisfied if:

Lossless

$$\Rightarrow \left.\begin{aligned} &|S_{21}|^2 + |S_{31}|^2 = 1 \\ &|S_{12}|^2 + |S_{32}|^2 = 1 \\ &|S_{13}|^2 + |S_{23}|^2 = 1 \\ &S_{31}^* \, S_{32} = 0 \\ &S_{21}^* \, S_{23} = 0 \\ &S_{12}^* \, S_{13} = 0 \end{aligned}\right\}$$

(1)

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

or

(2)

$$S_{21} = S_{32} = S_{13} = 0$$

$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

Note that $S_{ij} \neq S_{ji}$.

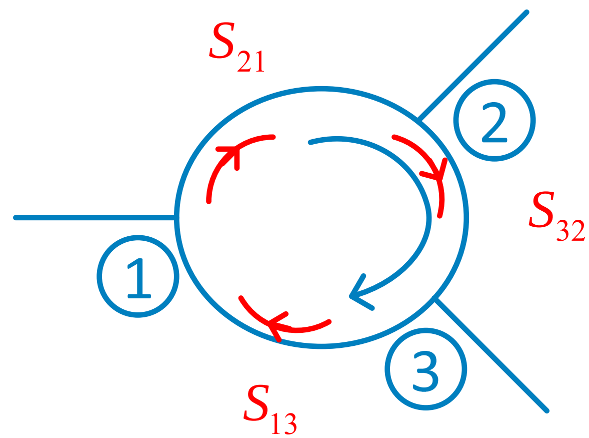
Circulators (cont.)

1

$$[S] = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

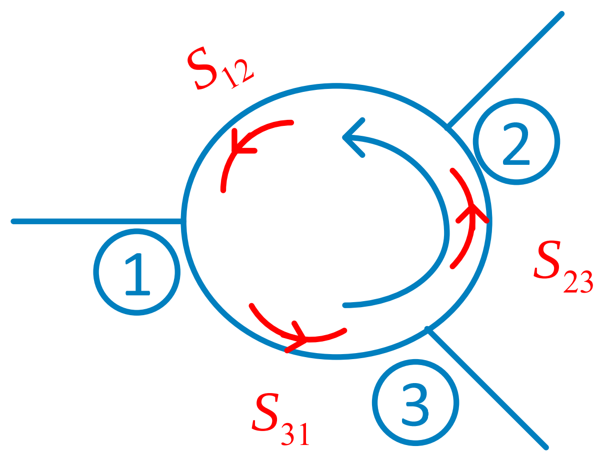
Note: We have assumed here that the phases of all the S parameters are zero.

Clockwise (LH) circulator



2

$$[S] = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

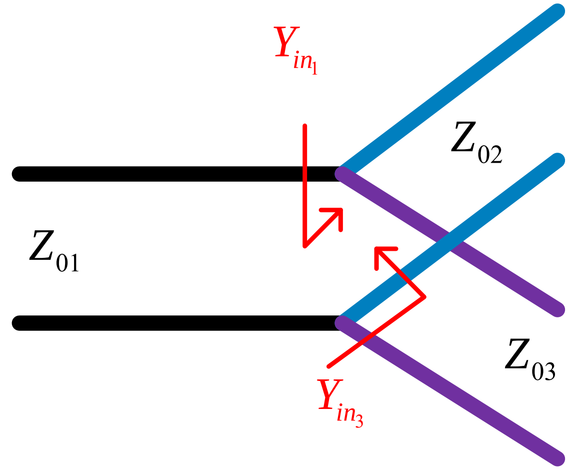


Counter-clockwise (RH) circulator

Circulators can be made using <u>biased ferrite</u> materials.

Power Dividers

T-Junction: lossless divider



$$Y_{in_1} = \frac{1}{Z_{02}} + \frac{1}{Z_{03}}$$

$$\text{To match:} \quad Z_{01} = Z_{02} \parallel Z_{03} = \frac{Z_{02} Z_{03}}{Z_{02} + Z_{03}}$$

$$\text{Note, however, } Y_{in_3} = \frac{1}{Z_{01}} + \frac{1}{Z_{02}} = \frac{Z_{02} + Z_{03}}{Z_{02} Z_{03}} + \frac{1}{Z_{02}} = \frac{Z_{02} + 2Z_{03}}{Z_{02} Z_{03}} = \frac{1}{Z_{03}} \left(\frac{Z_{02} + 2Z_{03}}{Z_{02}} \right)$$

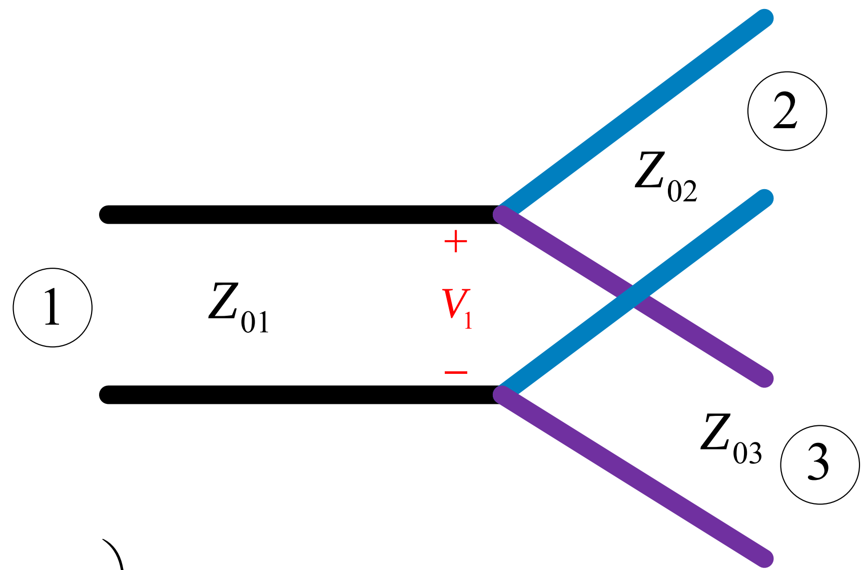
Thus, $Y_{in_3} \neq \dfrac{1}{Z_{03}}$ Also, $Y_{in_2} \neq \dfrac{1}{Z_{02}}$

If we match at port 1, we cannot match at the other ports!

Power Dividers (cont.)

Assuming port 1 matched:

$$Z_{01} = \frac{Z_{02} Z_{03}}{Z_{02} + Z_{03}}$$



$$P_{in_1} = \frac{1}{2} \frac{|V_1|^2}{Z_{01}}$$

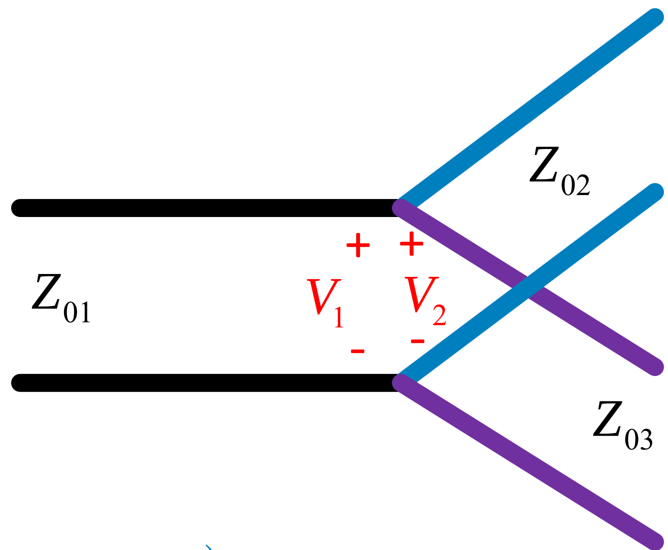
$$P_{out_2} = \frac{1}{2} \frac{|V_1|^2}{Z_{02}} = \frac{Z_{01}}{Z_{02}} P_{in_1} = \left(\frac{Z_{03}}{Z_{02} + Z_{03}} \right) P_{in_1} = K \, P_{in_1}$$

$$P_{out_3} = \frac{1}{2} \frac{|V_1|^2}{Z_{03}} = \frac{Z_{01}}{Z_{03}} P_{in_1} = \left(\frac{Z_{02}}{Z_{02} + Z_{03}} \right) P_{in_1} = (1 - K) \, P_{in_1}$$

We can design the splitter to control the powers into the two output lines.

Power Dividers (cont.)

Examine the reflection at each port (S_{ii}):



$$S_{11} = \left.\frac{V_1^- / \sqrt{Z_{01}}}{V_1^+ / \sqrt{Z_{01}}}\right|_{a_2=a_3=0} = \left.\frac{V_1^-}{V_1^+}\right|_{a_2=a_3=0}$$

$$= \frac{Z_{in1} - Z_{01}}{Z_{in1} + Z_{01}} = \frac{Z_{02} \| Z_{03} - Z_{01}}{Z_{02} \| Z_{03} + Z_{01}} \quad (\text{zero if port 1 is matched})$$

Note: A match on port 1 requires

$$Z_{02} > Z_{01}$$
$$Z_{03} > Z_{01}$$

(since the two output lines combine in parallel)

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{a_1=a_3=0} = \frac{Z_{01} \| Z_{03} - Z_{02}}{Z_{01} \| Z_{03} + Z_{02}}$$

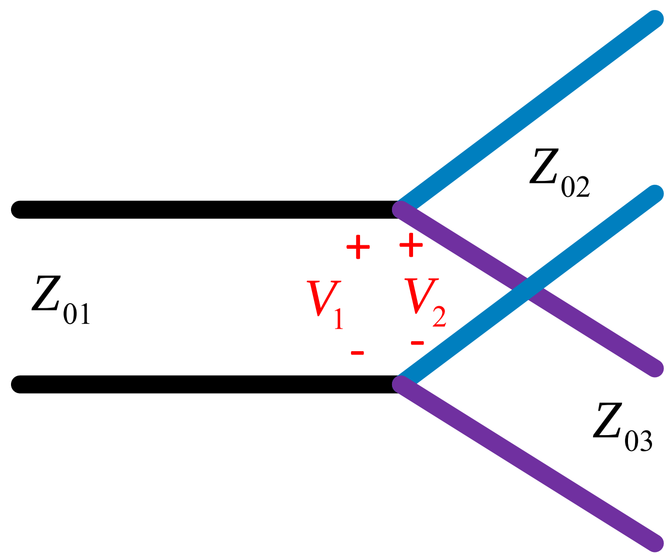
$$S_{33} = \left.\frac{V_3^-}{V_3^+}\right|_{a_1=a_2=0} = \frac{Z_{01} \| Z_{02} - Z_{03}}{Z_{01} \| Z_{02} + Z_{03}}$$

$$\Rightarrow S_{22} \neq 0, \quad S_{33} \neq 0$$

Power Dividers (cont.)

Also, we have:

$$S_{21} = \left. \frac{\dfrac{V_2^-}{\sqrt{Z_{02}}}}{\dfrac{V_1^+}{\sqrt{Z_{01}}}} \right|_{a_2 = a_3 = 0}$$



$$V_1 = V_1^+ \left(1 + S_{11}\right); \; V_2^- = V_2 = V_1 \quad \Rightarrow \quad V_2^- / \; V_1^+ = \; V_1 / V_1^+ = 1 + S_{11}$$

$$\Rightarrow S_{21} = \left(1 + S_{11}\right) \sqrt{\frac{Z_{01}}{Z_{02}}} = S_{12}$$

Similarly,

$$S_{31} = S_{13} = \left(1 + S_{11}\right) \sqrt{\frac{Z_{01}}{Z_{03}}} \text{ and } S_{32} = S_{23} = \left(1 + S_{22}\right) \sqrt{\frac{Z_{02}}{Z_{03}}}$$

Power Dividers (cont.)

If port 1 is matched: $Z_{01} = \dfrac{Z_{02} Z_{03}}{Z_{02} + Z_{03}}$

$$\Rightarrow S_{11} = 0 \;;\quad S_{22} = \frac{Z_{01} \| Z_{03} - Z_{02}}{Z_{01} \| Z_{03} + Z_{02}} \;;\quad S_{33} = \frac{Z_{01} \| Z_{02} - Z_{03}}{Z_{01} \| Z_{02} + Z_{03}}$$

$$S_{21} = S_{12} = \left(1 + S_{11}\right) \sqrt{\frac{Z_{01}}{Z_{02}}} = \sqrt{\frac{Z_{01}}{Z_{02}}} = \sqrt{\frac{Z_{03}}{Z_{02} + Z_{03}}}$$

$$S_{13} = S_{31} = \left(1 + S_{11}\right) \sqrt{\frac{Z_{01}}{Z_{03}}} = \sqrt{\frac{Z_{01}}{Z_{03}}} = \sqrt{\frac{Z_{02}}{Z_{02} + Z_{03}}}$$

$$S_{32} = S_{23} = \left(1 + S_{22}\right) \sqrt{\frac{Z_{02}}{Z_{03}}} = \left(1 + S_{22}\right) \sqrt{\frac{Z_{02}}{Z_{03}}}$$

$$[S] = \begin{pmatrix} 0 & S_{21} & S_{31} \\ S_{21} & S_{22} & S_{32} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

Only $S_{11} = 0$

The output ports are not isolated.

Power Dividers (cont.)

Output powers:

$$\frac{P_2}{P_1} = |S_{21}|^2 = \left(\frac{Z_{03}}{Z_{02} + Z_{03}} \right)$$

$$\frac{P_3}{P_1} = |S_{31}|^2 = \left(\frac{Z_{02}}{Z_{02} + Z_{03}} \right)$$

Note: P_1 is the input power on port 1.

Hence

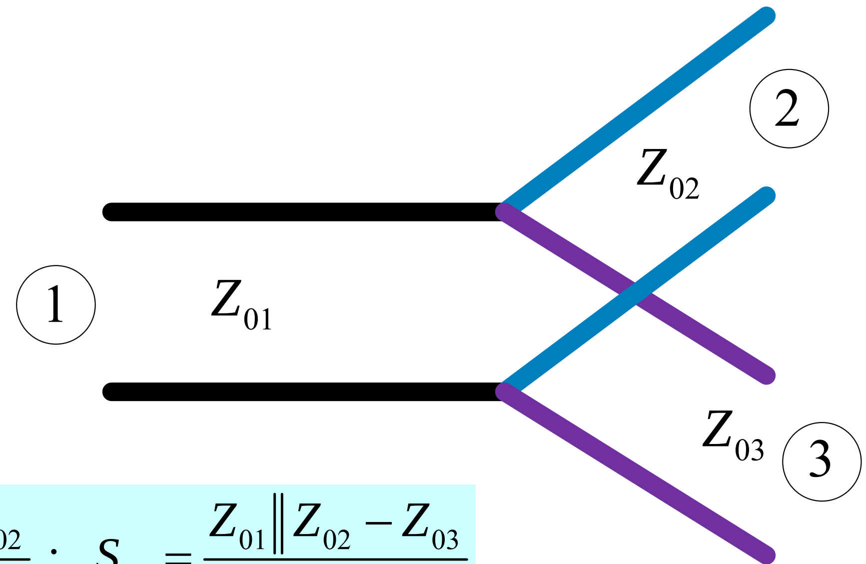
$$\frac{P_3}{P_2} = \frac{Z_{02}}{Z_{03}}$$

$$\text{Check}: \; P_1 = P_2 + P_3 = \left(\frac{Z_{03}}{Z_{02} + Z_{03}} \right) P_1 + \left(\frac{Z_{02}}{Z_{02} + Z_{03}} \right) P_1 = P_1$$

Power Dividers (cont.)

Summary

$$Z_{01} = \frac{Z_{02} Z_{03}}{Z_{02} + Z_{03}}$$



$$\frac{P_3}{P_2} = \frac{Z_{02}}{Z_{03}}$$

$$S_{11} = 0 \;;\quad S_{22} = \frac{Z_{01} \| Z_{03} - Z_{02}}{Z_{01} \| Z_{03} + Z_{02}} \;;\quad S_{33} = \frac{Z_{01} \| Z_{02} - Z_{03}}{Z_{01} \| Z_{02} + Z_{03}}$$

$$S_{21} = S_{12} = \sqrt{\frac{Z_{03}}{Z_{02} + Z_{03}}}$$

$$S_{13} = S_{31} = \sqrt{\frac{Z_{02}}{Z_{02} + Z_{03}}}$$

$$S_{32} = S_{23} = \left(1 + S_{22}\right)\sqrt{\frac{Z_{02}}{Z_{03}}}$$

- The input port is matched, but not the output ports.
- The output ports are not isolated.

⇩

Waves reflected from devices on ports 2 and 3 with cause interference with the devices.

Power Dividers (cont.)

Example: Microstrip T-junction power divider

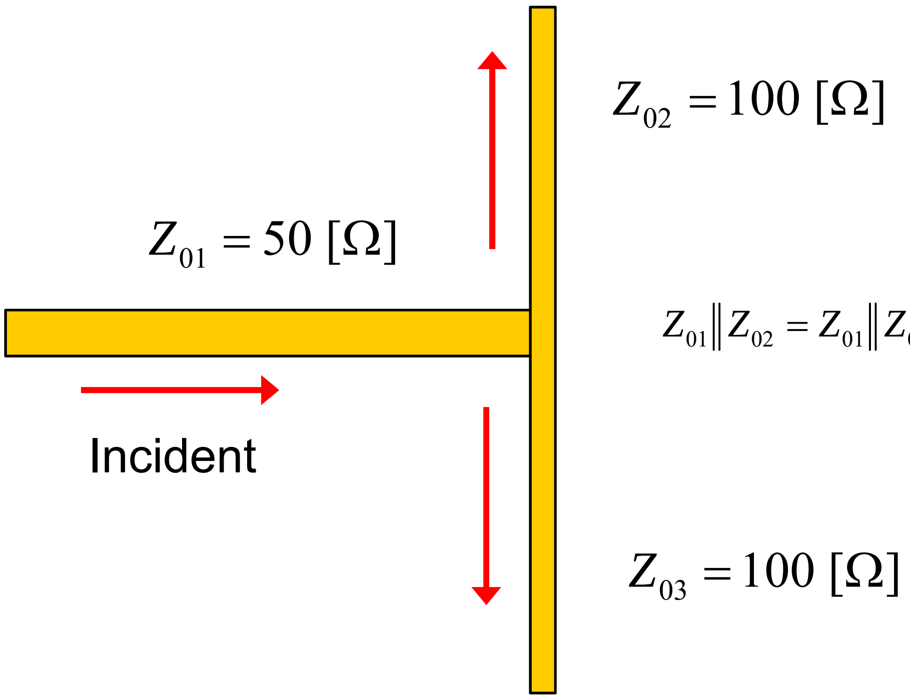
$$S_{11} = 0$$

$$S_{22} = S_{33} = -\frac{1}{2}$$

$$S_{21} = S_{12} = \sqrt{\frac{1}{2}}$$

$$S_{31} = S_{13} = \sqrt{\frac{1}{2}}$$

$$S_{32} = S_{23} = \frac{1}{2}$$



$$Z_{01} \| Z_{02} = Z_{01} \| Z_{03} = 50 \| 100 = 33.333\ [\Omega]$$

Power Dividers (cont.)

The matched power divider also works as a match power combiner

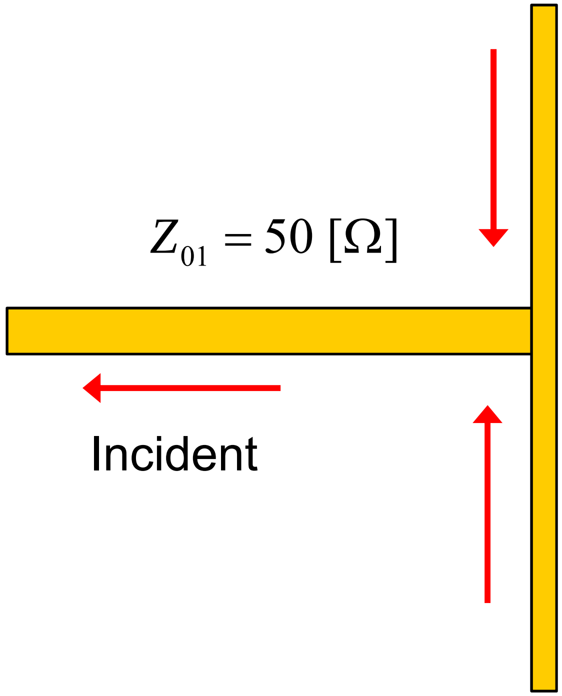
$$S_{11} = 0$$

$$S_{22} = S_{33} = -\frac{1}{2}$$

$$S_{21} = S_{12} = \sqrt{\frac{1}{2}}$$

$$S_{31} = S_{13} = \sqrt{\frac{1}{2}}$$

$$S_{32} = S_{23} = \frac{1}{2}$$



$$Z_{02} = 100\ [\Omega]$$

$$Z_{01} \| Z_{02} = Z_{01} \| Z_{03} = 50 \| 100 = 33.333\ [\Omega]$$

$$Z_{03} = 100\ [\Omega]$$

$$\begin{aligned} b_3 &= a_3 S_{33} + a_2 S_{32} \\ &= a_3 \left(S_{33} + S_{32} \right) \\ &= 0 \end{aligned}$$

Equal waves are incident from ports 2 and 3.

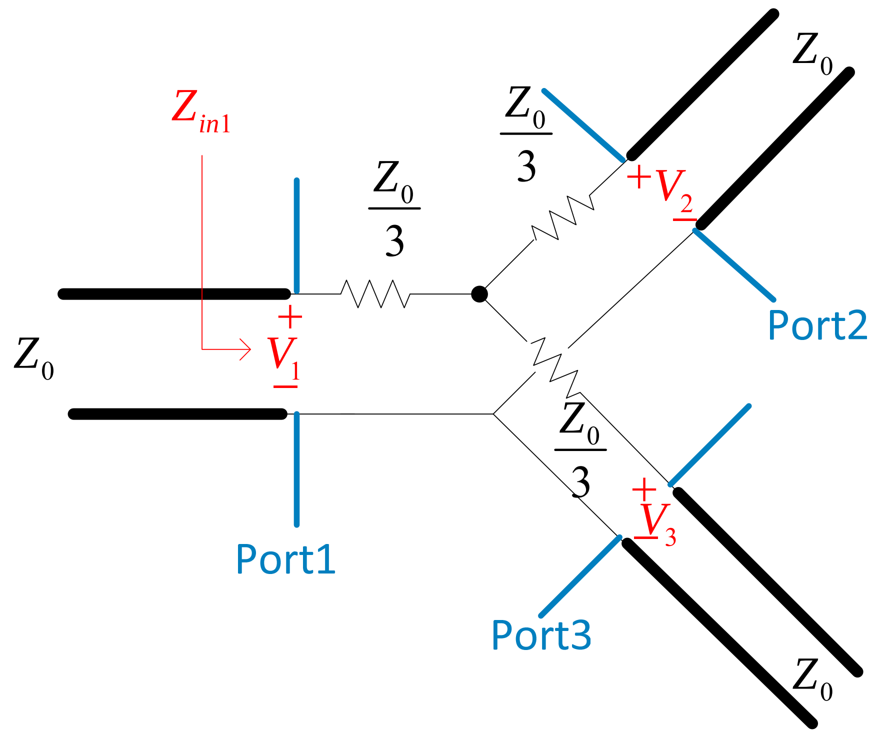
4/30/2018

Resistive Power Divider

$$Z_{in1} = \frac{Z_0}{3} + \frac{4Z_0}{3} \Big\| \frac{4Z_0}{3}$$

$$= \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0$$

(The same for Z_{in1} and Z_{in2}.)

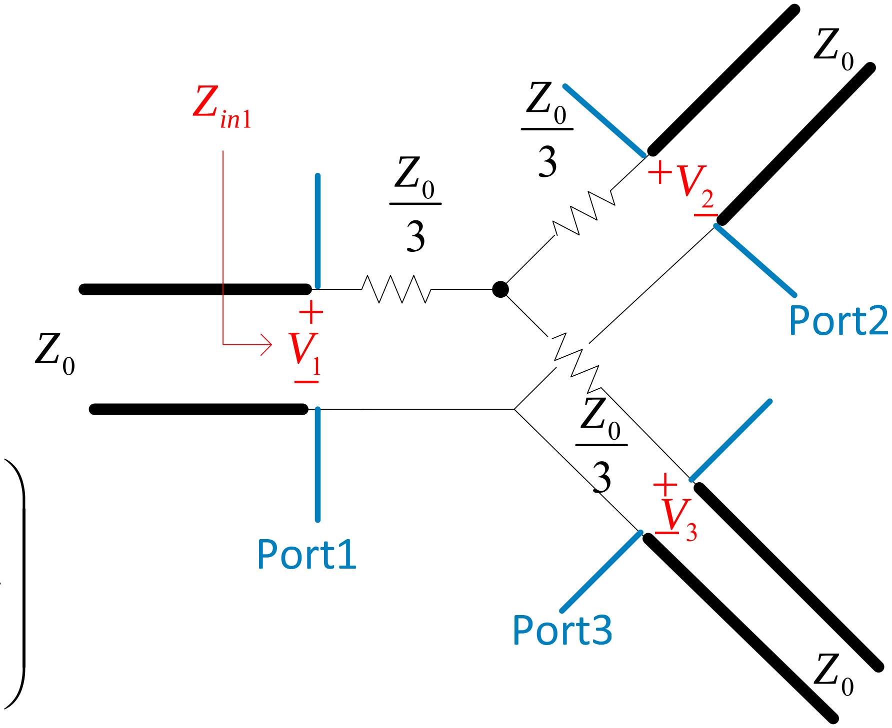


$\Rightarrow$ All ports are matched.

$$S_{11} = S_{22} = S_{33} = 0$$

Resistive Power Divider (cont.)

$$S_{21}=\left.\frac{\dfrac{V_2^-}{\sqrt{Z_0}}}{\dfrac{V_1^+}{\sqrt{Z_0}}}\right|_{a_2=a_3=0}$$



$$V_1=V_1^+\left(1+S_{11}\right)=V_1^+$$

$$V_2^-=V_2=V_1\left(\frac{\frac{2}{3}Z_0}{\frac{Z_0}{3}+\frac{2}{3}Z_0}\right)\left(\frac{Z_0}{\frac{Z_0}{3}+Z_0}\right)$$

$$=V_1^+\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)=\frac{1}{2}V_1^+$$

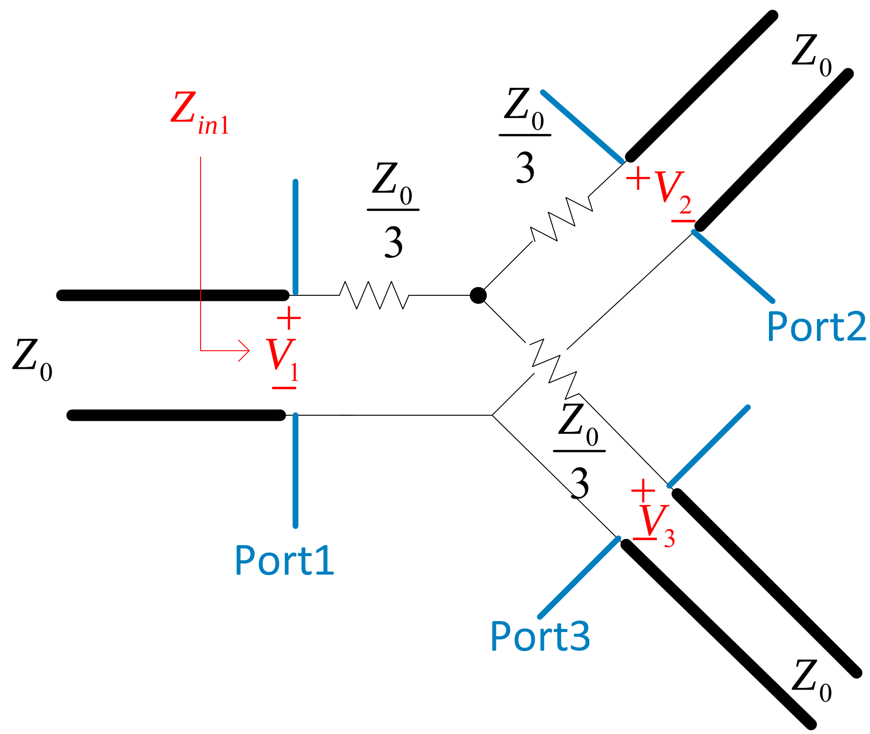
By reciprocity and symmetry

$$\Rightarrow S_{21}=\frac{1}{2}=S_{12}=S_{31}=S_{13}=S_{32}=S_{23}$$

Resistive Power Divider (cont.)

Hence we have

$$[S] = \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$



$$P_1 = P_{in} = \frac{1}{2}\frac{|V_1|^2}{Z_0} = \frac{1}{2}|a_1|^2$$

$$P_2 = P_3 = \frac{1}{2}|b_2|^2 = \frac{1}{2}|a_1 S_{21}|^2 = P_1|S_{21}|^2 = P_1\left|\frac{1}{2}\right|^2 = \frac{P_{in}}{4}$$

All ports are matched, but $1/2\ P_{in}$ is dissipated by resistors, and the output ports are not isolated.

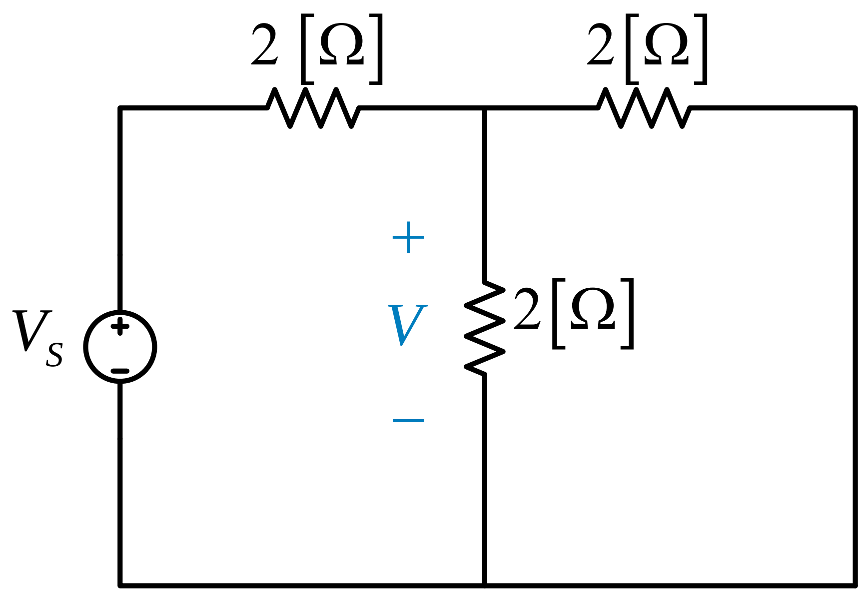
Even-Odd Mode Analysis

(This is needed for analyzing the Wilkinson)

Example: We want to solve for V.

We do this using even/odd mode analysis.

(This works because the circuit itself is symmetric.)

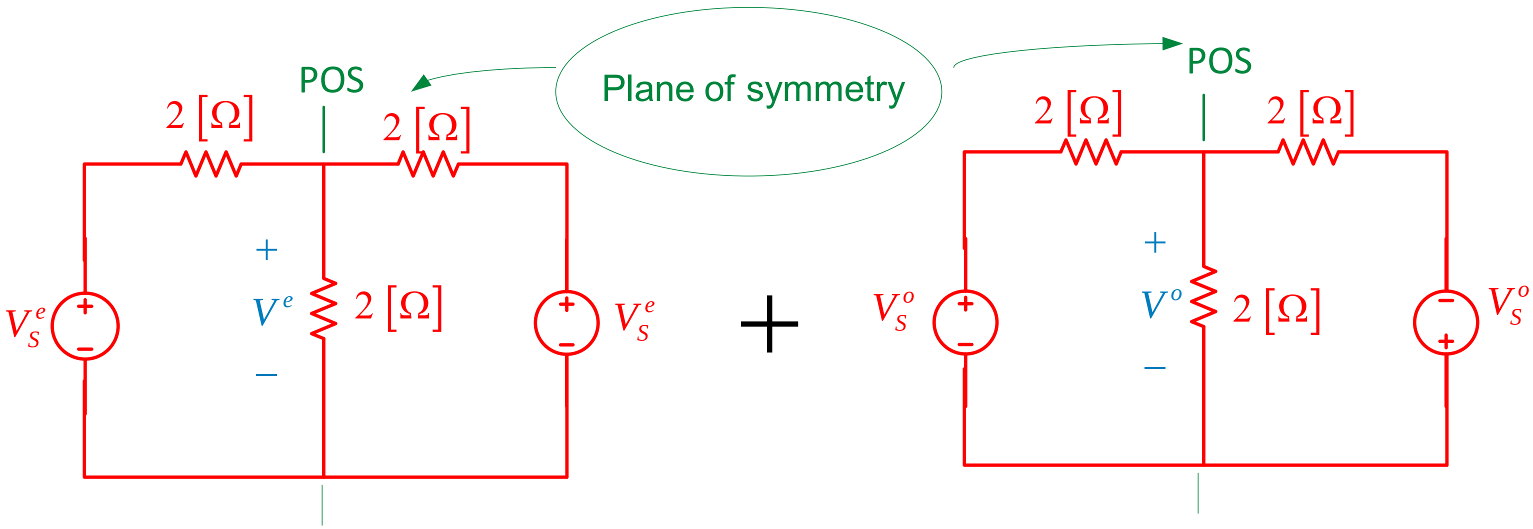


Correct answer :

$$V = V_S\left(\frac{1}{2+1}\right) = \frac{V_S}{3}$$

Even-Odd Mode Analysis

Let $V_S^e = V_S^o = \frac{1}{2} V_S$



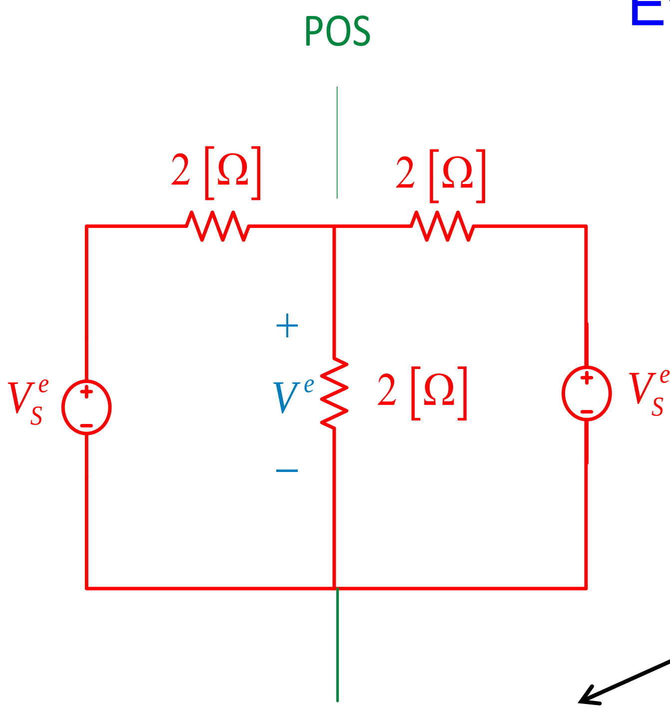
"Even" problem

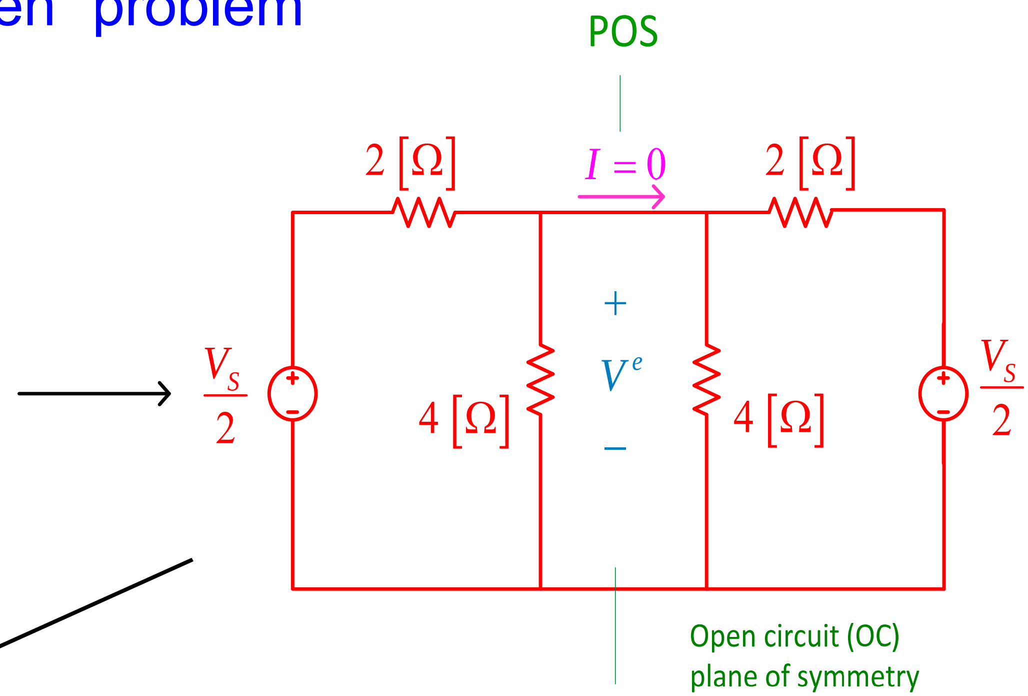
"Odd" problem

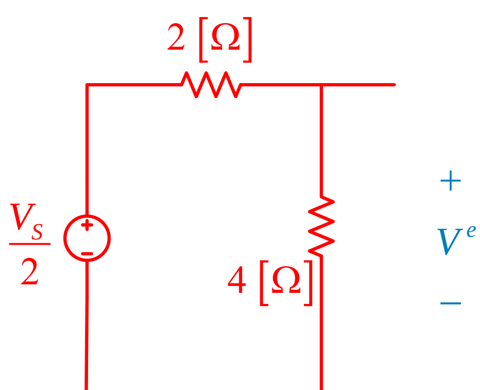
$$V = V^e + V^o$$

Even-Odd Mode Analysis (cont.)

“Even” problem



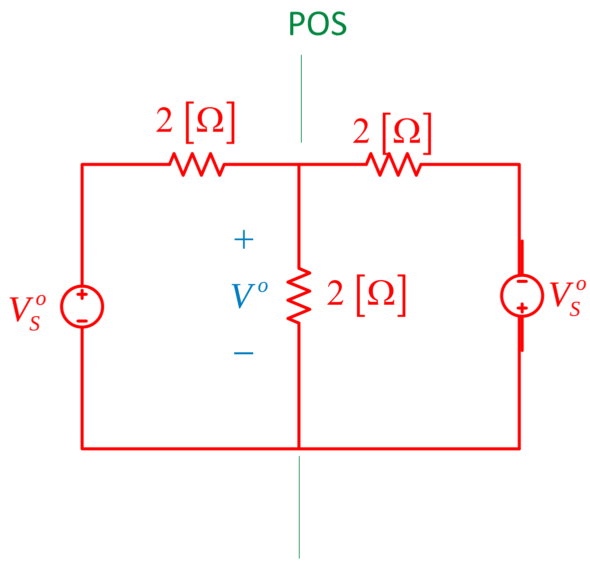


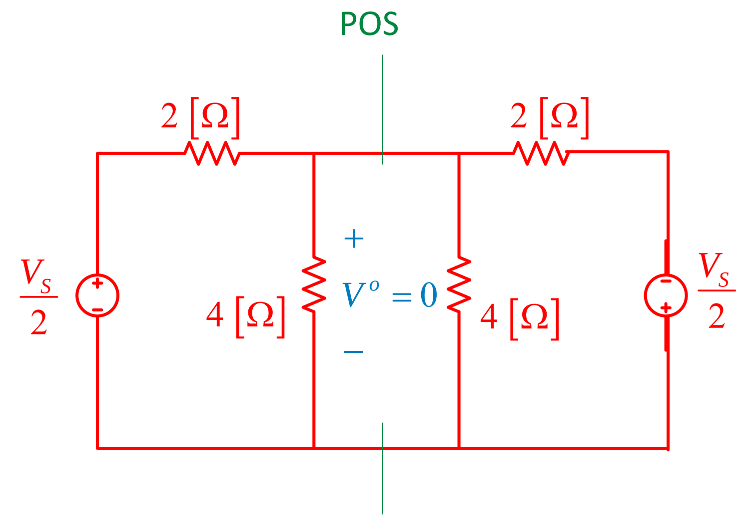


$$V^e = \frac{V_S}{2}\left(\frac{4}{2+4}\right) = \frac{V_S}{3}$$

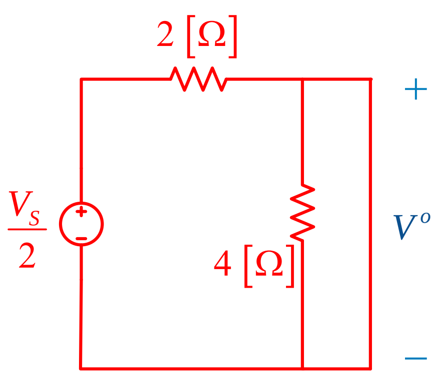
Even-Odd Mode Analysis (cont.)

"Odd" problem





Short circuit (SC) plane of symmetry



$$V^o = 0$$

Even-Odd Mode Analysis (cont.)

By superposition:

$$V = V^e + V^o = \frac{V_S}{3} + 0$$

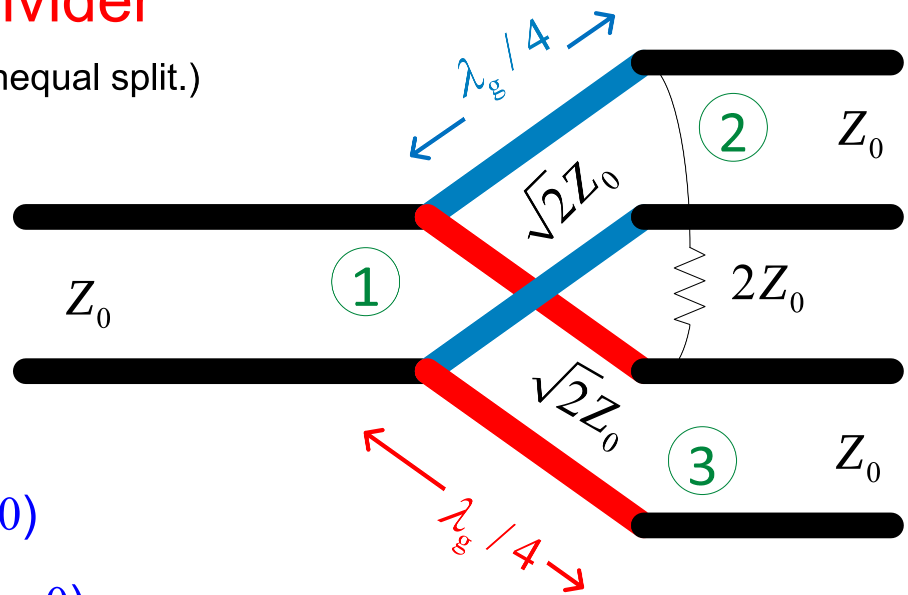
Hence we have

$$V = \frac{V_S}{3}$$

Wilkinson Power Divider

Equal-split (3 dB) power divider

(The Wilkinson can be designed to have an unequal split.)



- All ports matched ($S_{11} = S_{22} = S_{33} = 0$)
- Output ports are isolated ($S_{23} = S_{32} = 0$)

Note: No power is lost in going from port 1 to ports 2 and 3.

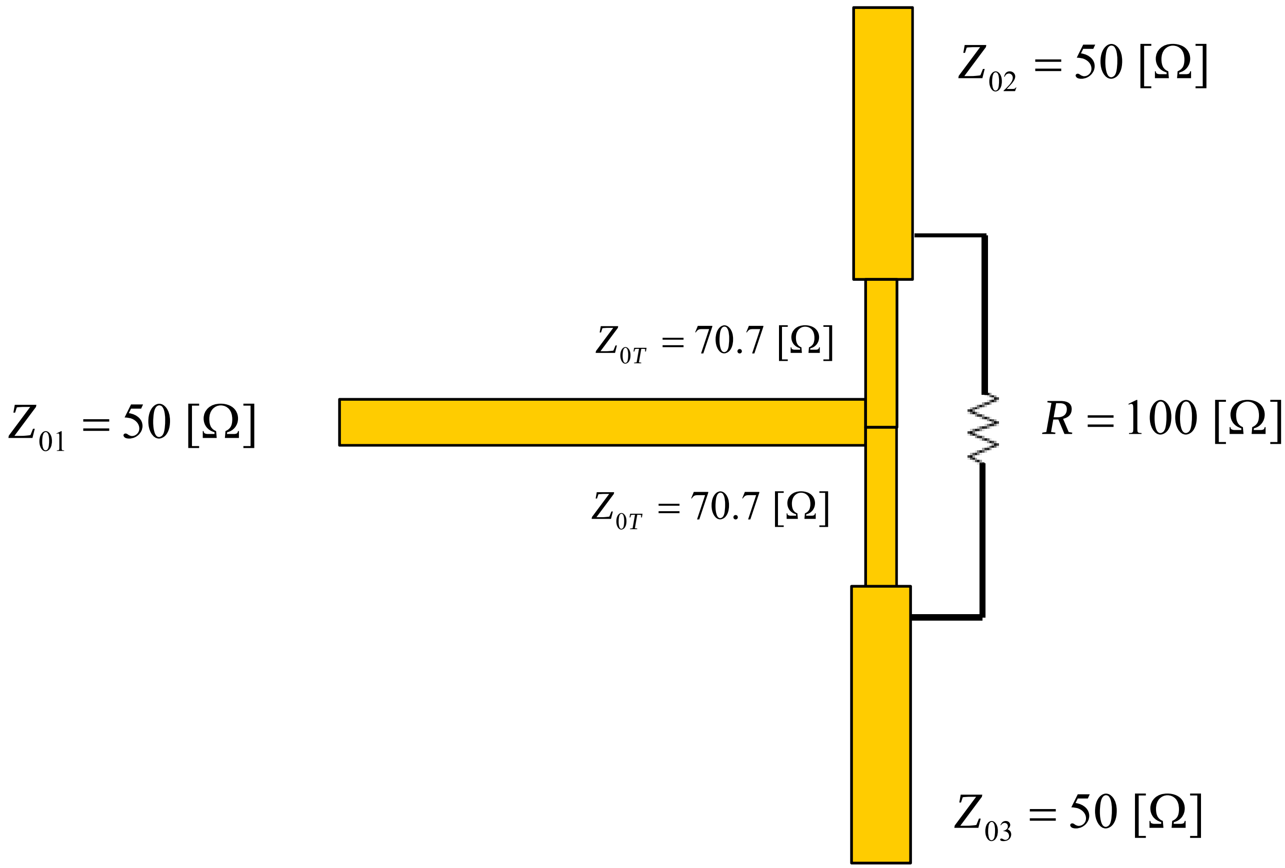
$$|S_{21}|^2 = |S_{31}|^2 = \frac{1}{2}$$

$$[S] = \frac{-j}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Obviously not unitary

Wilkinson Power Divider (cont.)

Example: Microstrip Wilkinson power divider



Wilkinson Power Divider (cont.)

- Even and odd analysis is used to analyze the structure when port 2 is excited.

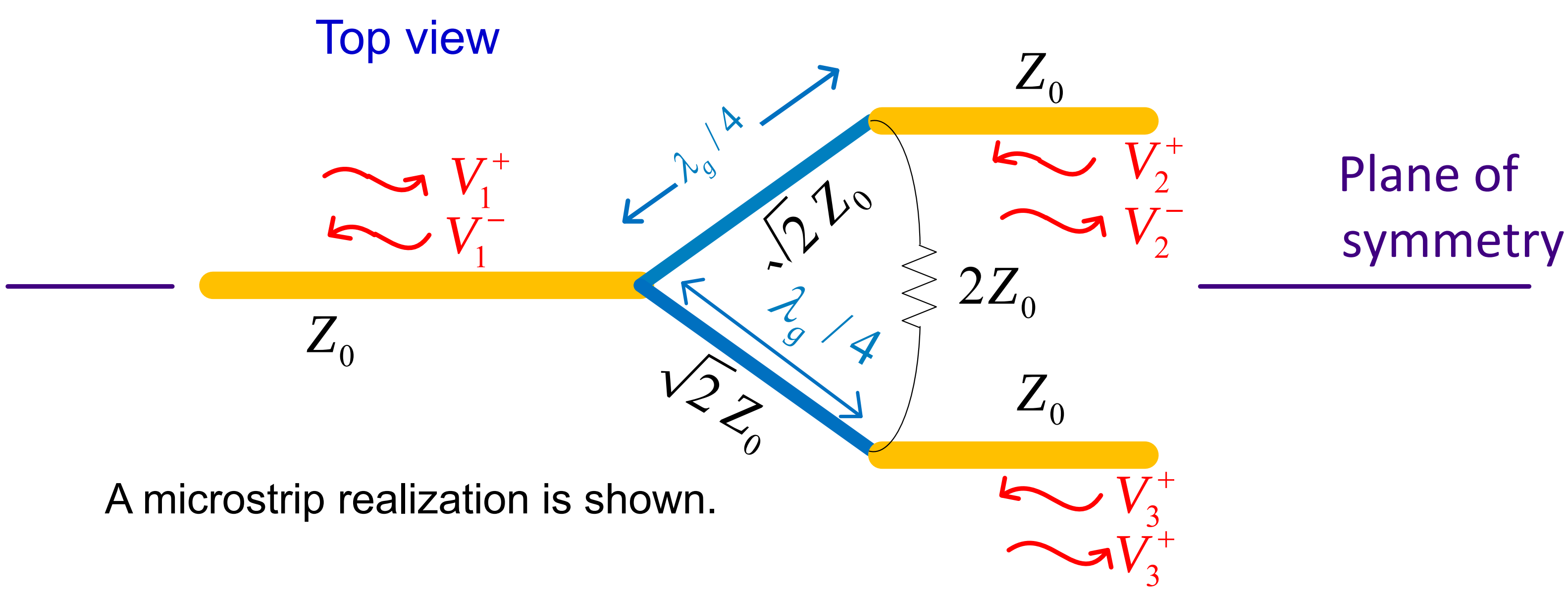
$\Rightarrow$ To determine S_{22}, S_{32}

- Only even analysis is needed to analyze the structure when port 1 is excited.

$\Rightarrow$ To determine S_{11}, S_{21}

The other components can be found by using symmetry and reciprocity.

Wilkinson Power Divider (cont.)

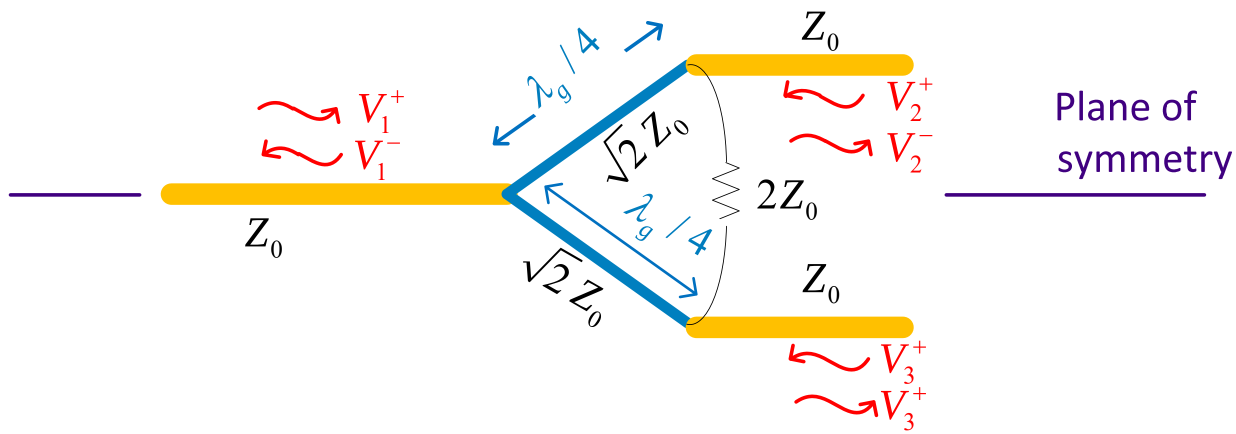


A microstrip realization is shown.

Split structure along plane of symmetry (POS)

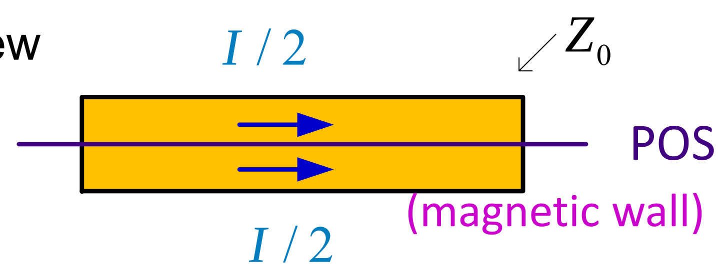
Even $\Rightarrow$ voltage even about POS $\Rightarrow$ place OC along POS
Odd $\Rightarrow$ voltage odd about POS $\Rightarrow$ place SC along POS

Wilkinson Power Divider (cont.)



How do you split a transmission line? (This is needed for the even case.)

Top view



Z_0 microstrip line

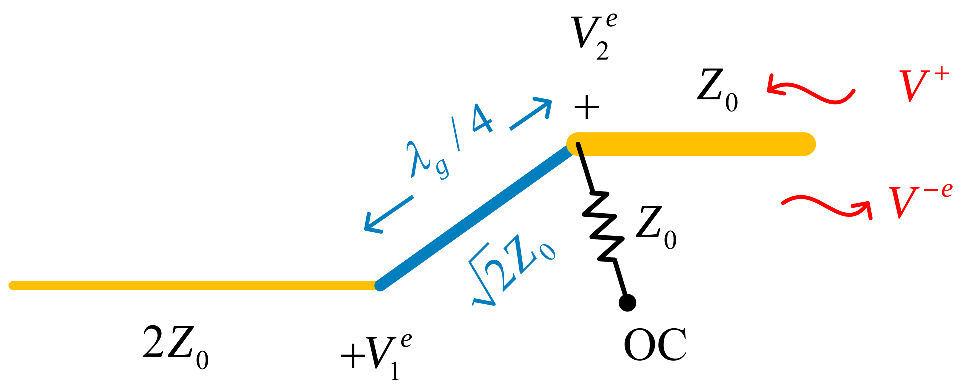
Voltage is the same for each half of line (V)
Current is halved for each half of line ($I/2$)

$$\Rightarrow Z_0^h = \frac{V}{I/2} = 2Z_0$$

For each half

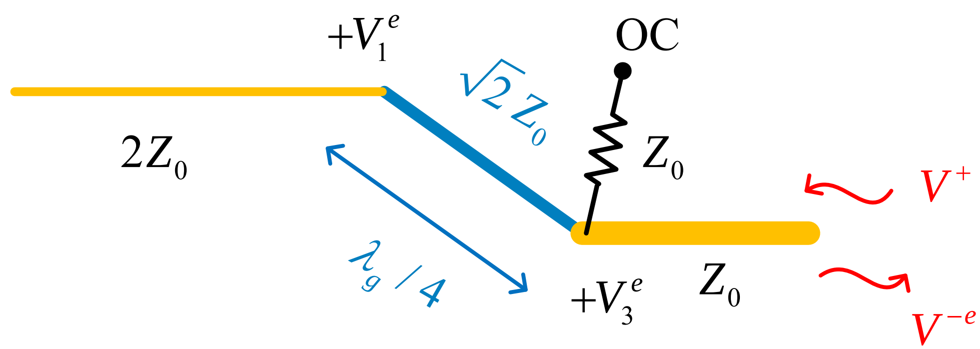
Wilkinson Power Divider (cont.)

"Even" Problem



Note: The $2Z_0$ resistor has been split into two Z_0 resistors in series.

Ports 2 and 3 are excited in phase.

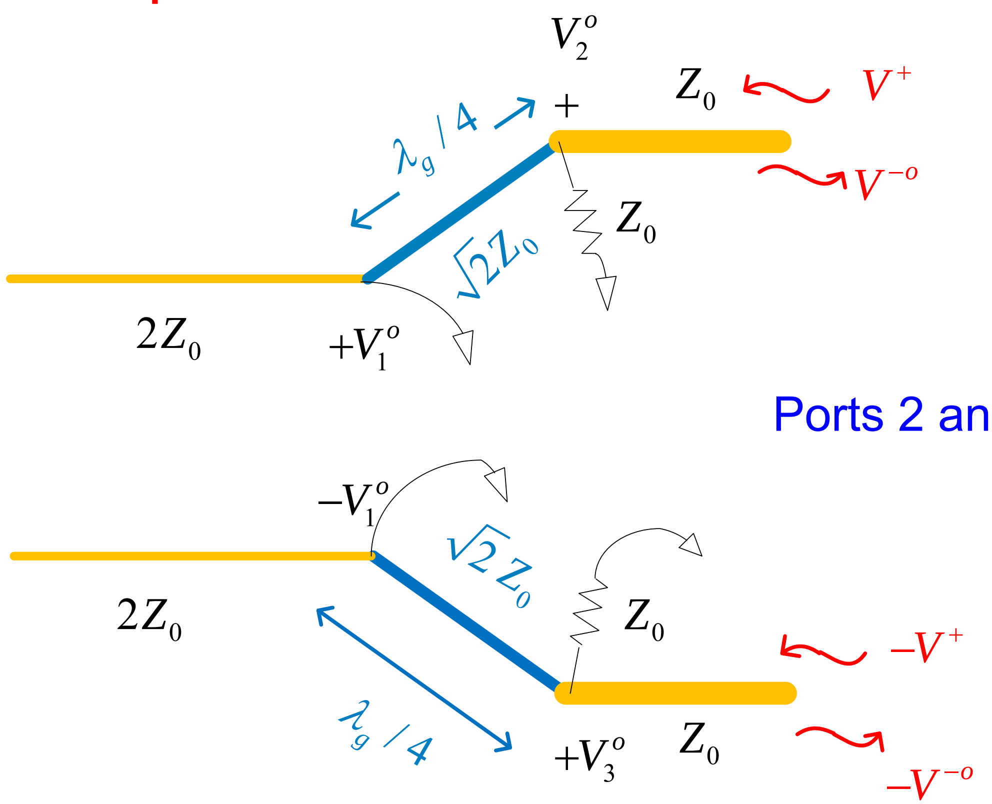


$$\text{Note}: \quad V_3^e = V_2^e$$

Wilkinson Power Divider (cont.)

"Odd" problem

Note: The $2Z_0$ resistor has been split into two Z_0 resistors in series.



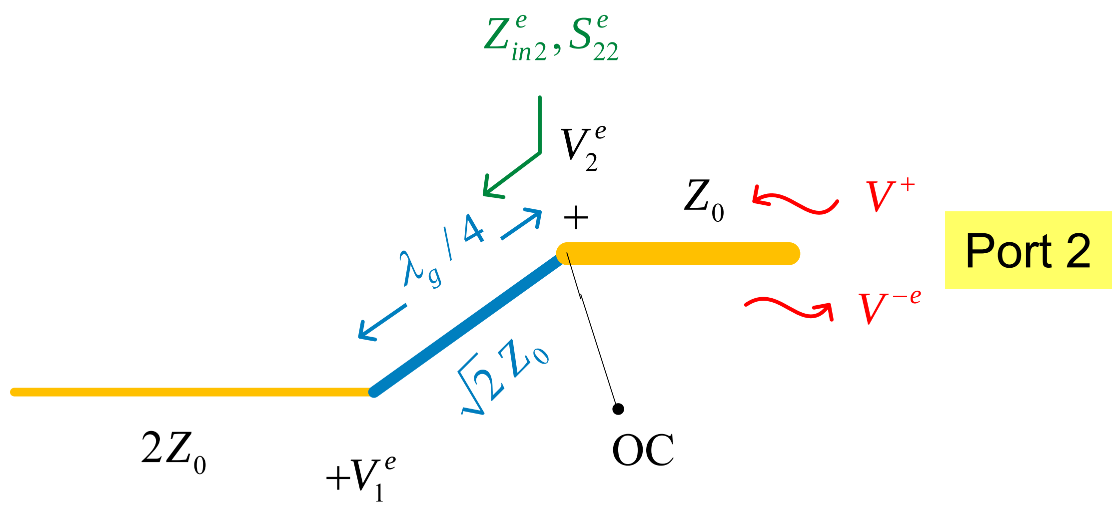
Ports 2 and 3 are excited 180° out of phase.

$$\text{Note: } V_1^o = 0, \quad V_3^o = -V_2^o$$

Wilkinson Power Divider (cont.)

Even Problem

Port 2 excitation



$$Z_{in2}^e = \frac{\left(\sqrt{2}\, Z_0\right)^2}{2Z_0} = Z_0$$

$$\Rightarrow S_{22}^e = \frac{Z_{in2}^e - Z_0}{Z_{in2}^e + Z_0} = 0$$

Recall:

$$Z_{in} = \frac{Z_T^2}{Z_L}$$

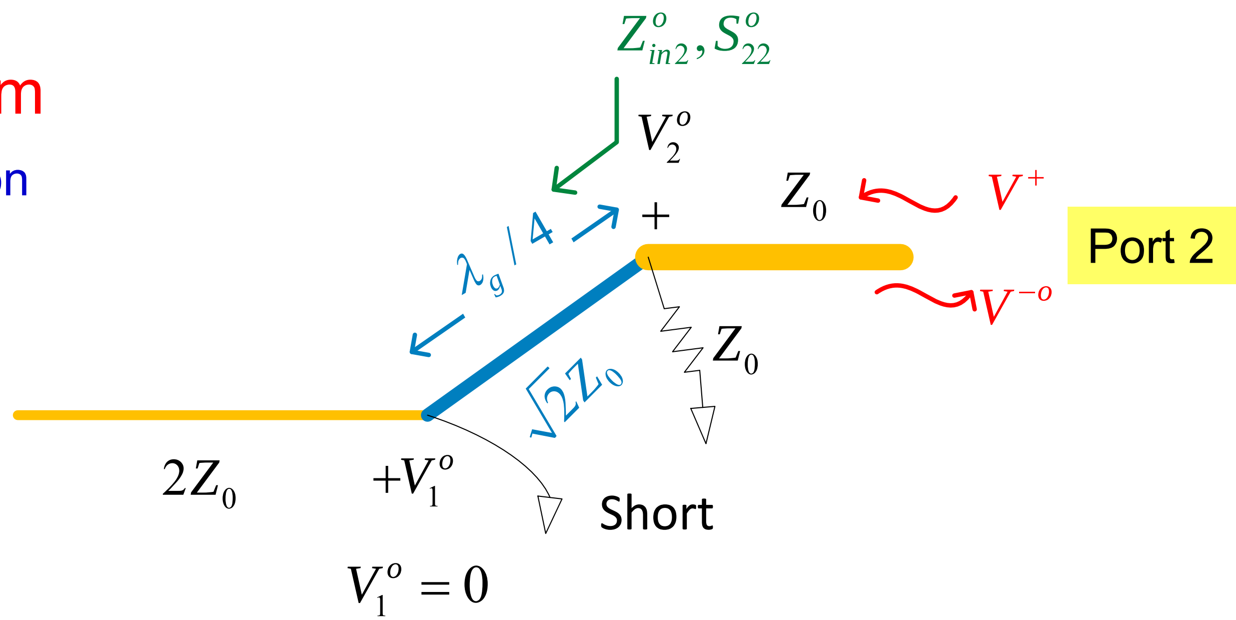
(quarter-wave transformer)

Also, by symmetry, $S_{33}^e = 0$

Wilkinson Power Divider (cont.)

Odd Problem

Port 2 excitation



$$Z_{in2}^o = \infty \| Z_0 = Z_0$$

$$\Rightarrow S_{22}^o = \frac{Z_{in2}^o - Z_0}{Z_{in2}^o + Z_0} = 0$$

Also, by symmetry, $S_{33}^o = 0$

Wilkinson Power Divider (cont.)

We add the results from the even and odd cases together:

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{a_1=a_3=0} \qquad S_{22} = \frac{V^{-e}+V^{-o}}{V^+ + V^+} = \frac{V^{-e}+V^{-o}}{2V^+} = \frac{1}{2}\left(S_{22}^e + S_{22}^o\right) = \frac{1}{2}(0+0) = 0$$

$$\Rightarrow \quad S_{33} = 0$$ (by symmetry)

$$S_{32} = \left.\frac{V_3^-}{V_2^+}\right|_{a_1=a_3=0} \qquad S_{32} = \frac{V^{-e}-V^{-o}}{V^+ + V^+} = \frac{V^{-e}-V^{-o}}{2V^+} = \frac{1}{2}\left(S_{22}^e - S_{22}^o\right) = \frac{1}{2}(0-0) = 0$$

$$\Rightarrow \quad S_{23} = 0$$ (by reciprocity)

In summary, for port 2 excitation, we have:

$$S_{22} = 0$$

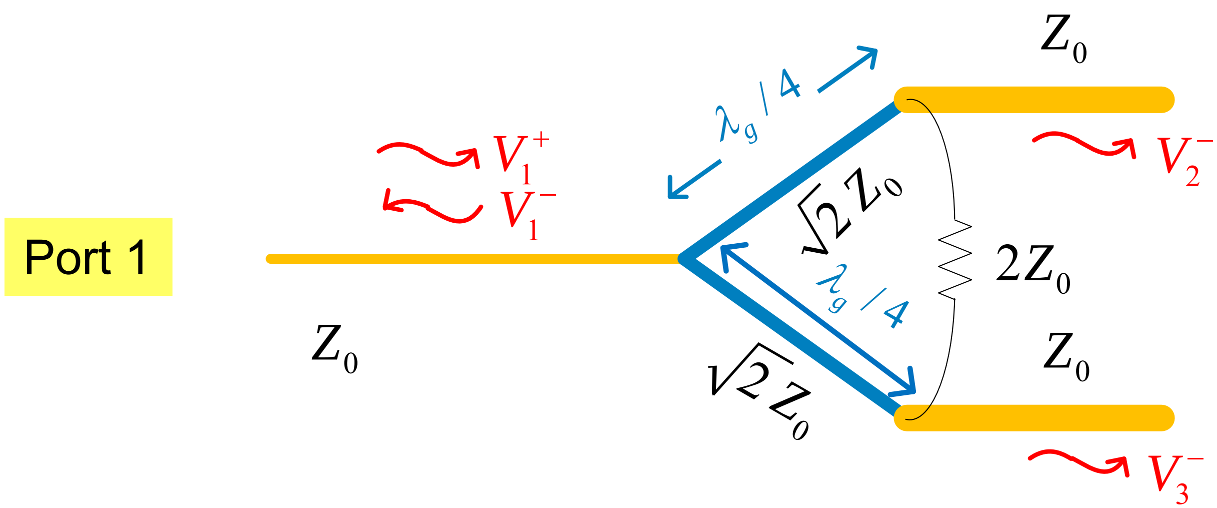
$$S_{33} = 0$$

$$S_{32} = S_{23} = 0$$

Note: Since all ports have the same Z_0, we ignore the normalizing factor $\sqrt{Z_0}$ in the S parameter definition.

Wilkinson Power Divider (cont.)

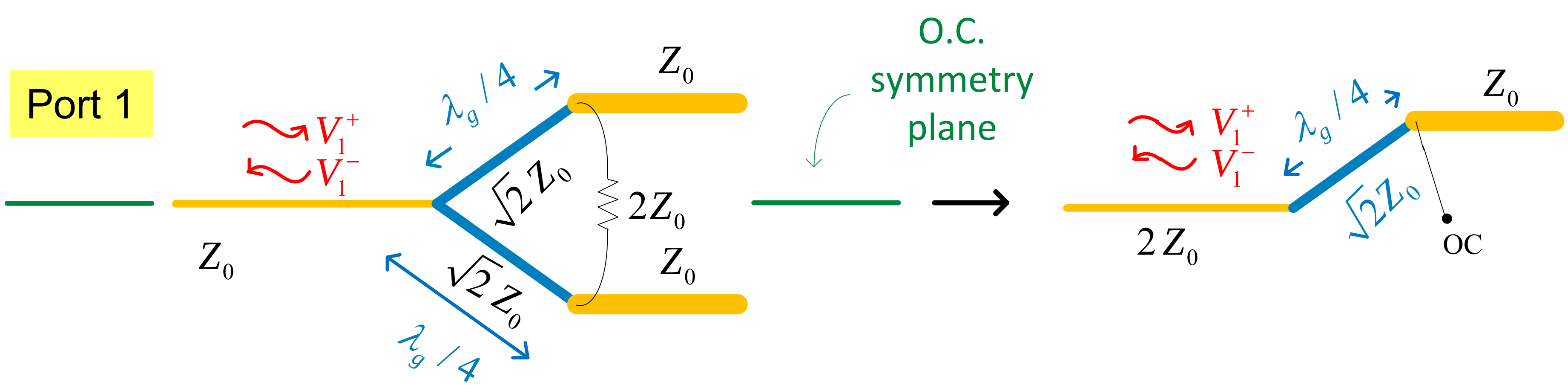
Port 1 excitation



When port 1 is excited, the response, by symmetry, is even.
(Hence, the total fields are the same as the even fields.)

Wilkinson Power Divider (cont.)

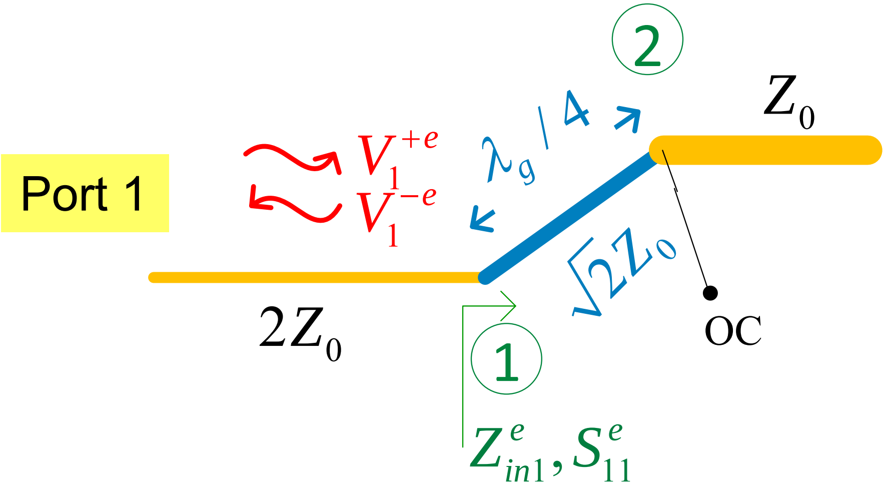
Even Problem



Wilkinson Power Divider (cont.)

Even Problem

Port 1 excitation



$$Z_{in1}^e = \frac{\left(\sqrt{2}Z_0\right)^2}{Z_0} = 2Z_0$$

$$S_{11}^e = \frac{Z_{in1}^e - 2Z_0}{Z_{in1}^e + 2Z_0} = 0$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{a_2=a_3=0} = \left.\frac{V_1^{-e}}{V_1^{+e}}\right|_{a_2=0} = S_{11}^e = 0$$

Hence

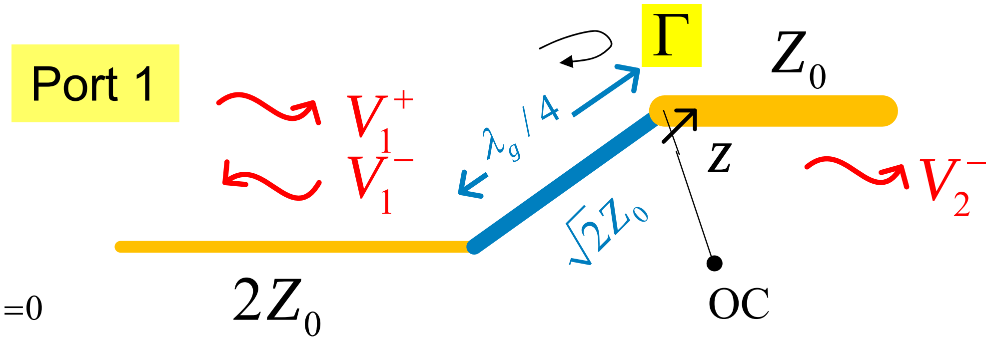
$$S_{11} = 0$$

Wilkinson Power Divider (cont.)

Even Problem

Port 1 excitation

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{a_2 = a_3 = 0}$$



$$V_1 = V_1^+ \left(1 + \cancel{S_{11}}\right) = V_1^+$$

$$V_2 = V_2^- + \cancel{V_2^+}$$

Along $\lambda_g/4$ wave transformer:

$$V(z) = V_0^+ e^{-j\beta z}\left(1 + \Gamma e^{+j2\beta z}\right)$$

$z =$ distance from port 2

$$\begin{cases} V_2 = V(0) = V_0^+ (1 + \Gamma) \\ V_1 = V(-\lambda_g / 4) = V_0^+ j (1 - \Gamma) \end{cases}$$

$$\Rightarrow S_{21} = \frac{V_2^-}{V_1^+} = \frac{V_2}{V_1} = -j \frac{(1+\Gamma)}{(1-\Gamma)} = -j \frac{2}{2\sqrt{2}}$$

$$\Gamma = \frac{Z_0 - \sqrt{2} Z_0}{Z_0 + \sqrt{2} Z_0} = \frac{1 - \sqrt{2}}{1 + \sqrt{2}}$$

$$\Rightarrow \boxed{S_{21} = \frac{-j}{\sqrt{2}} = S_{12}}$$

(reciprocal)

Wilkinson Power Divider (cont.)

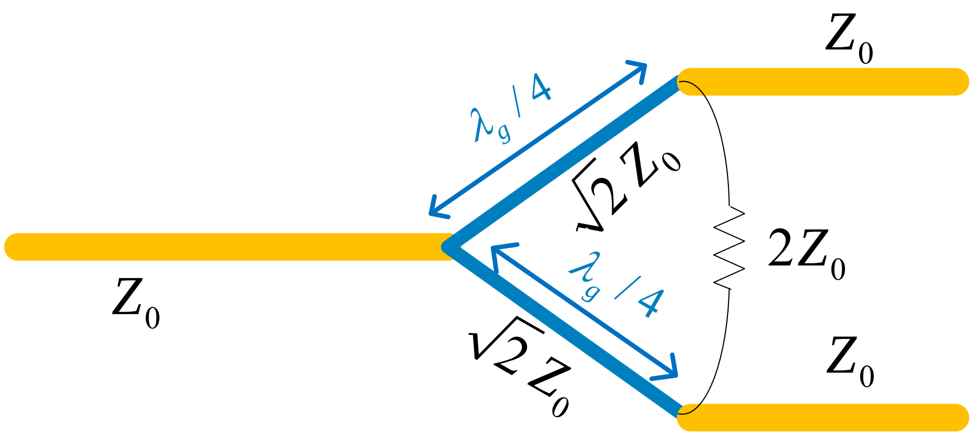
For the other components:

By symmetry: $$S_{31} = S_{21} = \frac{-j}{\sqrt{2}}$$

By reciprocity: $$S_{13} = S_{31} = \frac{-j}{\sqrt{2}}$$

Wilkinson Power Divider (cont.)

$$[S] = \frac{-j}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



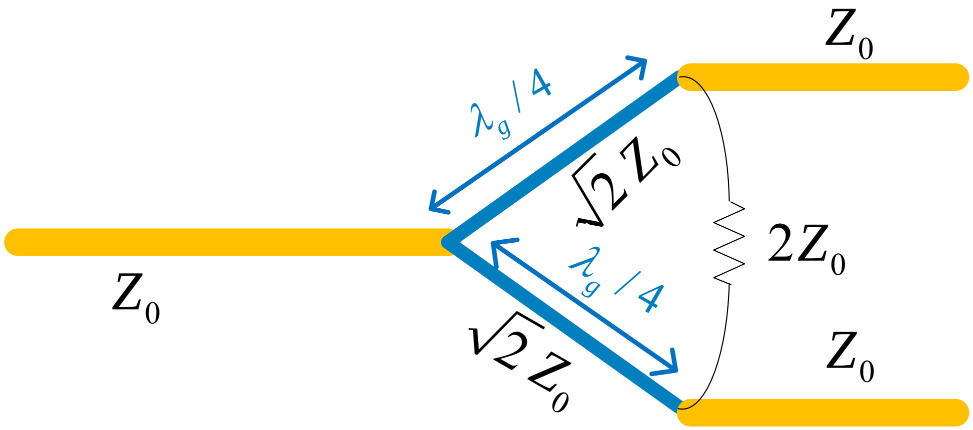
$$S_{11} = S_{22} = S_{33} = 0$$

$$S_{32} = S_{23} = 0$$

All three ports are matched, and the output ports are isolated.

Wilkinson Power Divider (cont.)

$$[S] = \frac{-j}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



$$S_{21} = S_{31} = \frac{-j}{\sqrt{2}}$$

$$S_{12} = S_{13} = \frac{-j}{\sqrt{2}}$$

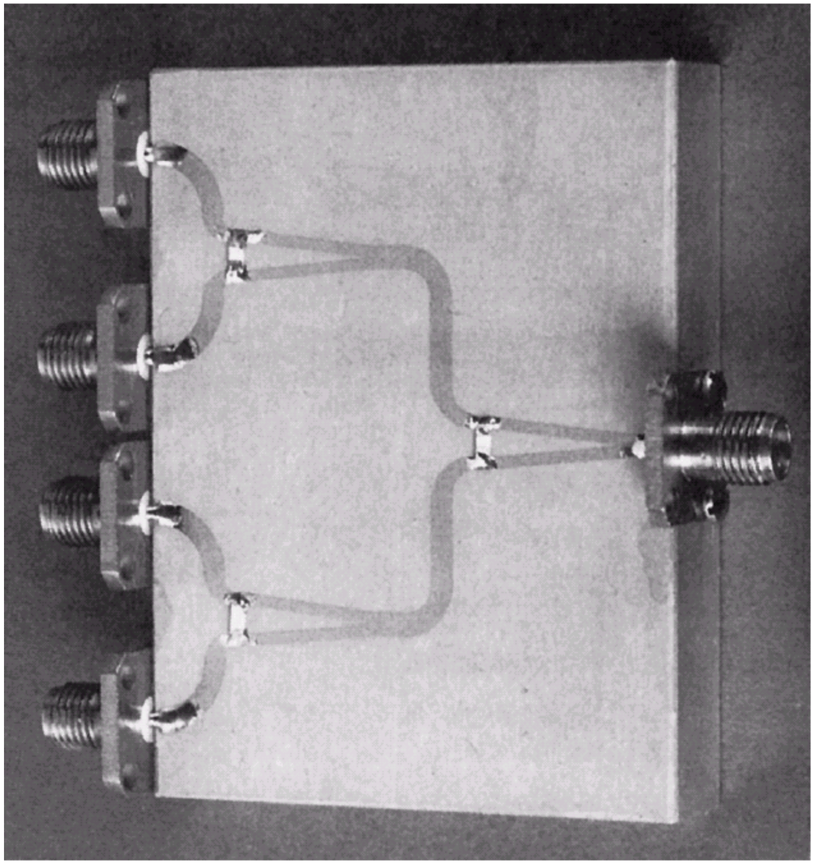
- When a wave is incident from port 1, half of the total incident power gets transmitted to each output port (no loss of power).
- When a wave is incident from port 2 or port 3, half of the power gets transmitted to port 1 and half gets absorbed by the resistor, but nothing gets through to the other output port.

Wilkinson Power Divider (cont.)

Figure 7.15 of Pozar

Photograph of a four-way corporate power divider network using three microstrip Wilkinson power dividers. Note the isolation chip resistors.

Courtesy of M.D. Abouzahra, MIT Lincoln Laboratory.



Wilkinson Power Divider (cont.)

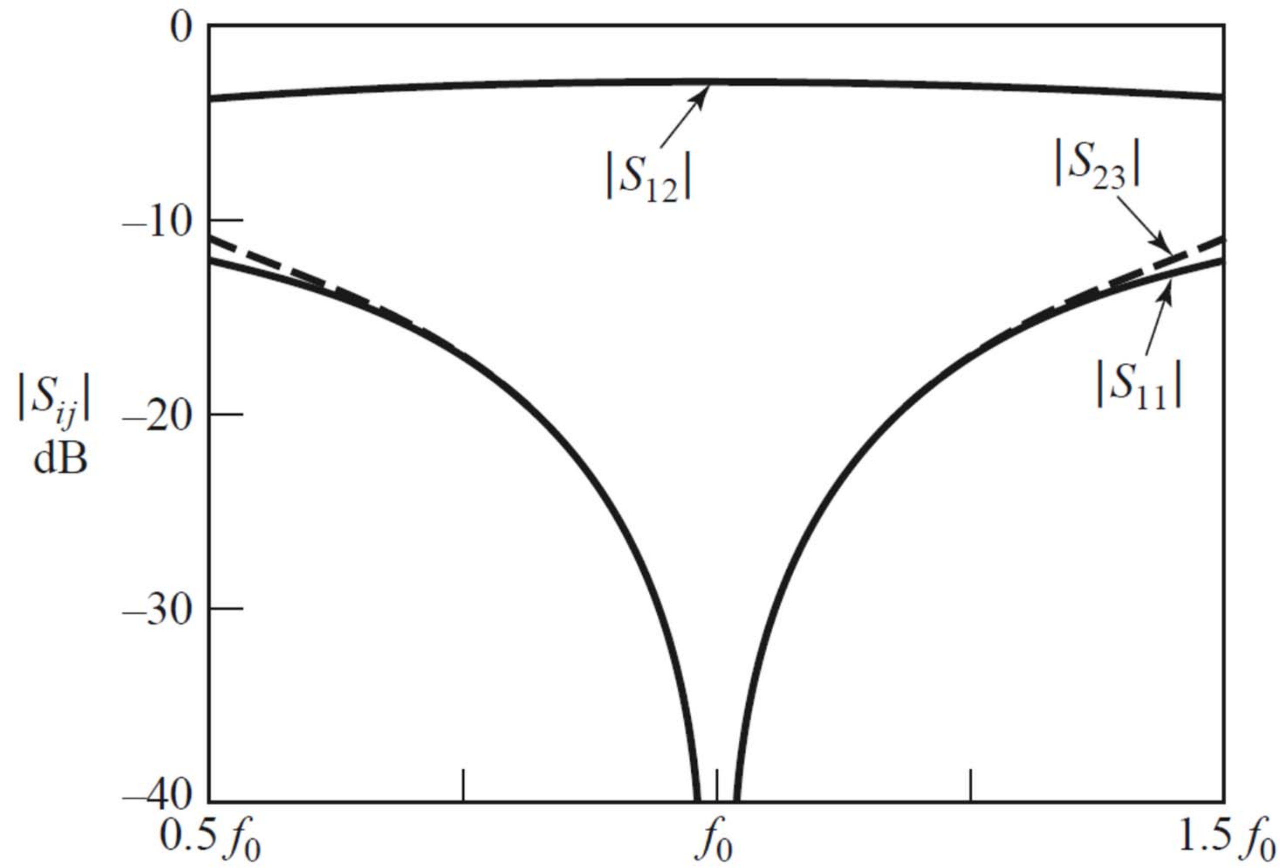


Figure 7.12 of Pozar

Frequency response of an equal-split Wilkinson power divider. Port 1 is the input port; ports 2 and 3 are the output ports.